efficients of the tangential stresses with those of normal stresses one observes that for lengthwise extension depending on the values of λ one has the inequality $\rho_\sigma < \rho_\tau$ for angles less than 40–45°. Similar effect is observed for extensions in the crosswise direction though now for angles > 40–45°. For $\alpha = 0$ the computational results agree with the known results [5].

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DETERMINATION OF THE RATE OF EVOLUTION OF ELASTIC ENERGY FOR A Γ-SHAPED CRACK BY THE METHOD OF MEASUREMENT OF THE PLIABILITY

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UDC 539.375

At the present time, the values of the stress-intensity coefficients and the rate of evolution of elastic energy are well known for various configurations of a body with rectilinear cracks [1]. For a nonrectilinear crack, the only known problem is that of an arc-shaped crack in an infinite sheet with homogeneous elongation in an arbitrary direction [2].

To clarify the character of the propagation of cracks in laminar materials, it is important to know the rate of evolution of elastic energy for Γ-shaped cracks, where, after the breakdown of an element of the matrix, peeling-off starts in the composite material. The theoretical solution of such a problem is rather complex and, up to the present time, has not been carried through. In the present article, this problem is solved experimentally by the method of measurement of the pliability.

A method for determining the rate of evolution of elastic energy from the change in the pliability with an increase in the length of the crack was proposed a long while ago [3]; however, we know of no work where the method has been implemented in practice. This is obviously connected with the necessity of making extremely exact measurements, which is difficult to do practically. In the present work, the method of measuring the frequency of the intrinsic vibrations was used, which makes it possible to measure the pliability of a sample with an accuracy up to 0.05%.

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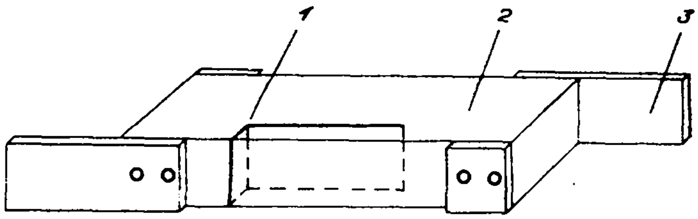

Fig. 1

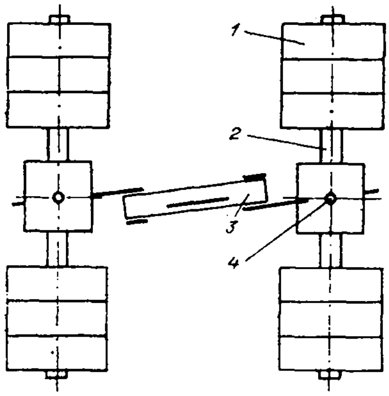

Fig. 2

We shall give the derivation of the principal formulas of the method, modified for the case of a body loaded by a bending moment. The energy V stored in the body is expressed in the form

$$V = (1/2)\alpha M,$$

where α is the angle of rotation of a cross section; M is the bending moment. If W is the total energy in the external loading connections and in the deformed body, then

$$dW/dl \equiv bG = M d\alpha/dl - dV/dl, \tag{1}$$

where G is the rate of evolution of elastic energy per unit area of the crack; l is the length of the crack; and b is the width of the sample. Assuming the body to be linearly elastic, we have

$$\alpha = \lambda M,$$

where λ is the value of the pliability of the body, which depends on the configuration, including the dimension, of the crack.

Differentiating the first and second terms in (1), we obtain

$$d\alpha/dl = d(\lambda M)/dl = \lambda dM/dl + M d\lambda/dl,$$

$$\frac{dV}{dl} = \frac{d\left(\frac{1}{2}\lambda M^2\right)}{dl} = \frac{1}{2}M^2\frac{d\lambda}{dl} + M\lambda\frac{dM}{dl};$$

substituting into (1), we obtain

$$G = (M^2/2b)d\lambda/dl.$$

We introduce, in addition, the value of the rate of evolution of elastic energy with a unit bending moment

$$g = \frac{G}{M^2} = \frac{1}{2b}\frac{d\lambda}{dl}.$$

In the experiment, the value of the pliability λ was measured for different lengths of the crack l, and graphical differentiation was then carried out with respect to l. The measurements were made in samples of PKhV-1 foam plastic with transverse cross sections of 17 × 30, 25 × 45, and 38 × 68 mm. The crack was sawed in the samples with a jigsaw. The configuration of a crack is shown in Fig. 1 (1 is a Γ-shaped crack; 2 is the sam-

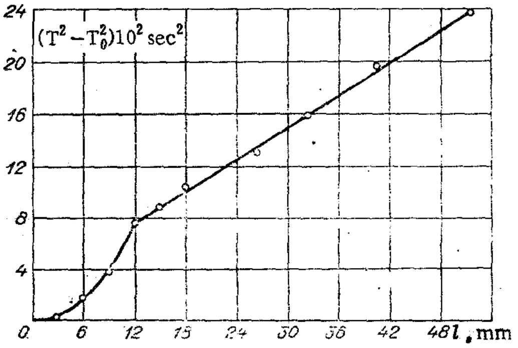

Fig. 3

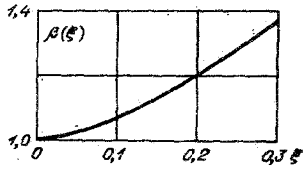

Fig. 4

ple; 3 is the holder). The length of the initial section of the crack, perpendicular to the edge, is 1/4 of the thickness of the sample; the crack then turns by 90° and passes parallel to the edge.

The pliability of the samples was measured from the frequency of the natural bending vibrations in the unit, a scheme of which is shown in Fig. 2 (1 are weights; 2 are bars; 3 is the sample; 4 are the points of suspension). To the rods there are attached strain gauges, connected with an amplifier and an F-5080 frequency meter. Measurements were made of the time interval, equal to 10 periods of the vibrations of the rods. The period of the free vibrations is given by the formula

$$T = 2\pi\sqrt{\lambda J/2},$$

where J is the moment of inertia of each of the rods, or

$$\lambda = T^2/2\pi^2 J.$$

Figure 3 gives the dependence of T^2 on the length of the crack l for one of the samples tested. There can be seen a rapid rise in the slope of the curve in the initial section, corresponding to an increase in the rate of evolution of elastic energy. After the turn in the crack, the slope decreases and changes only very little with the further growth of the crack.

A comparison between the experimental and theoretical data can be made in the initial section before the turn of the crack and for a sufficiently long peeling crack, where $l \gg h$. For an edge crack in a semiinfinite sample, the stress-intensity coefficient is given by the formula [3]

$$K_{\mathrm{I}} = 1.12\sigma\sqrt{\pi l}, \quad G = \frac{K_{\mathrm{I}}^2}{\mathrm{E}} = \frac{1.12^2\pi\sigma^2 l}{\mathrm{E}},$$

where σ is the elongation stress. With the bending of a sufficiently thick sample, where

$$l/h \ll 1, \ \sigma = 6M/bh^2, \tag{2}$$

$$G = 1.12^2\pi 36 l M^2/\mathrm{E}(bh^2)^2,$$

respectively,

$$g = 1.12^2\pi 36 l/\mathrm{E}(bh^2)^2. \tag{3}$$

If condition (2) is not satisfied, then the correction factor β (l/h) must be introduced into formula (3), i.e.,

$$g = (1.12^2\pi 36 l/\mathrm{E}(bh^2)^2)\beta(l/h).$$

The value of this factor was calculated on the basis of the results of [1] and is given in Fig. 4.

If we construct the dependence of g and λ on the dimensionless parameter $\xi = l/$h, then

$$\frac{d\lambda}{d\left(\frac{l}{h}\right)} = 2bhg = \frac{1.12^2\pi\, 36\cdot 2}{\mathrm{E}}\,\frac{l}{h}\,\frac{1}{bh^2}\,\beta\left(\frac{l}{h}\right),$$

or

$$\mathrm{E}bh^2 d\lambda/d\xi = 2\cdot 1.12^2\pi 36\xi\beta(\xi). \tag{4}$$

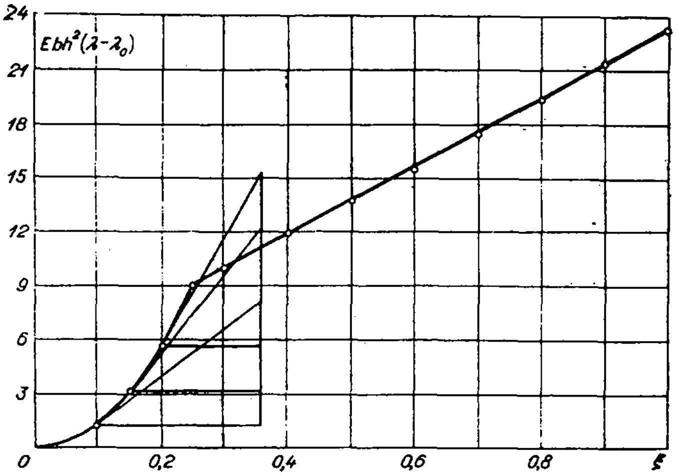

Fig. 5

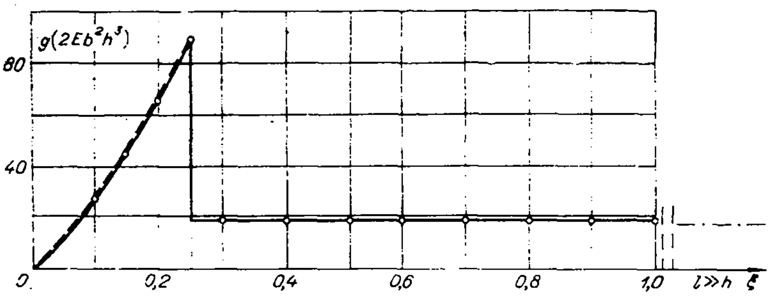

Fig. 6

Figure 5 shows the dependence of the value of $Ebh^2(\lambda - \lambda_0)$ on ξ, averaged over eight samples (λ_0 is the value of λ in the absence of a notch). The calculations were made using the value $E/J = 5.7 \cdot 10^8\ \mathrm{sec}^{-2} \cdot \mathrm{m}^{-3}$, obtained from measurements in samples without a notch. Figure 6 gives the derivative of the solid curve of Fig. 5. Differentiation was carried out graphically. In Fig. 6 the dashed line shows the theoretical curve for the initial section before the turn of the crack, corresponding to the left-hand side of Eq. (4). There is good agreement between the curves, coinciding with an accuracy up to the errors in the measured value of E (the elastic modulus), used in plotting the experimental curves.

The second section in which a comparison can be made between the experimental and theoretical results is the section of a long peeling crack $l_{\text{peel}} \gg h$. As is shown in [4], in this section the rate of evolution of the energy is constant,

$$g = \frac{6}{Eb^2}\left(\frac{2}{(h - l_\perp)^3} - \frac{1}{h^3}\right).$$

For the form of the crack used, when $l_\perp/h = 0.25$, $g = 8.25/Eb^2h^3$,

$$Ebh^2 d\lambda/d\xi = Ebh^3 d\lambda/dl = Ebh^3 2bg = 16.5.$$

The corresponding straight line is shown by the dashed-dot line in the right-hand part of Fig. 6. The most interesting section of the curve of the dependence of $d\lambda/d\xi$ on ξ is the section immediately after the turn of the crack. At the turning point of the crack, there is a jumpwise decrease in the value of g and, as can be seen from Fig. 6, with a further increase in the peeling crack, the value of g changes only insignificantly.

The value of G, the rate of evolution of elastic energy, behaves in the same manner, differing from g only by the factor M^2, i.e., $G/G_\perp = 4.7$ (G is the rate of evolution of elastic energy before the turn of the crack and $G_\perp$, after the turn of the crack).

Although these results were obtained for a definite geometry of the sample, the value of $G_\perp/G = 0.21$ obtained is obviously universal for the case where, for the original crack, only one coefficient of the intensity of the stresses K_I differs from zero. For the appearance of a Γ-shaped crack, it is obvious that the two coefficients K_I and K_{II} differ from zero; these cannot be determined separately by the method used. However, with an analysis of the conditions for the propagation of a crack, the value of the rate of evolution of elastic energy itself, which is determined directly in the given method, is important. The method can be applied to bodies of complex form with cracks also having a complex form.

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SOLUTION IN THE FORM OF SERIES OF LEGENDRE POLYNOMIALS OF AN AXISYMMETRIC MIXED PROBLEM FOR A HOLLOW ELASTIC CYLINDER

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UDC 539.3

In this paper it is shown that the method proposed in [1] for the solution of a plane mixed problem of the theory of elasticity can be used also for the solution of an axisymmetric mixed problem for a hollow elastic cylinder.

1. Statement of the Problem

In the case of axisymmetric elastic deformation the equations of equilibrium and Hooke's law can be written in the form

$$\frac{\partial p}{\partial x}+\frac{\partial \tau}{\partial y}+\gamma_1=0,\quad \frac{\partial \tau}{\partial x}+\frac{\partial q}{\partial y}-t+\gamma_2=0,$$
$$p-r\left(\lambda\varepsilon+2\mu\frac{\partial u}{\partial x}\right)=0,\quad q-r\left(\lambda\varepsilon+2\mu\frac{\partial v}{\partial y}\right)=0,$$
$$\tau-\mu r\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0,\quad t-\lambda\varepsilon-2\mu\frac{v}{r}=0,$$

where

$$\varepsilon=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{v}{r},\quad \lambda=\frac{\nu E}{(1+\nu)(1-2\nu)},\quad 0\leqslant\nu<\frac{1}{2},\quad E,\mu>0,$$
$$p=r\sigma_z,\ q=r\sigma_r,\ \tau=r\sigma_{rz},\ t=\sigma_\varphi,\ u=u_z,\ v=u_r,$$
$$x=z-z_0,\ y=r-r_0;$$

r, φ, and z are the cylindrical coordinates; σ_r, σ_φ, σ_z, σ_{rz}, u_r, and u_z are the components of the stress tensor and the displacement vector in the cylindrical coordinate system; γ_1 and γ_2 are the mass forces; z_0 and r_0 are constants; E is Young's modulus; μ is the shear modulus; and ν is Poisson's ratio.

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